RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. THIRD SEMESTER EXAMINATION, DECEMBER 2013

SECOND YEAR

Date : 18/12/2013 **STATISTICS (General)**

Time : 11 am – 1 pm Paper : III Full Marks : 50

1. Answer **any five** questions of the following :

a) Define a χ^2 (chi square) distribution. State its density function and find its mean. [4]

b) A random sample of size one is taken from exponential distribution with mean $\frac{1}{\theta}$, to test $H_0 : \theta = 2$ against $H_1 : \theta = 1$. Find the probabilities of type-I and type-II errors of the test : Reject H_0 iff $x \geq 1$. [4]

c) i) Define a randomized test.
ii) State the Neyman-pearson Fundamental lemma. [2+2]

d) If $\{T_n\}$ is a sequence of estimators such that $E(T_n) \to \theta$ and $Var(T_n) \to 0$ as $n \to \infty$, then show that T_n is consistent for θ. [4]

e) Let X_1, X_2 be iid as Poison with parameter λ. Show that $(X_1 + 2X_2)$ is not sufficient for λ. [4]

f) Consider a random sample of size 10 from a population with median m. Find $P[X_{(3)} \leq m \leq X_{(8)}]$, where $X_{(r)}$ denotes the rth order statistic. [4]

g) Show that for a sample of size two from $N(\mu, \sigma^2)$, the expected value of the sample range is $2\sigma/\sqrt{\pi}$. [4]

h) If $X \sim Bin(n, p)$, find an unbiased estimator of $p(1 - p)$. [4]

2. Answer **any three** questions of the following :

a) Show that in sampling from normal population the sample mean and the sample variance are independently distributed. Find the sampling distributions. [5+5]

b) Derive the ML estimator of μ and σ^2 of a normal distribution. Check that the estimator of μ is unbiased. Find the standard error of the estimator. [3+2+5]

c) i) What are variance stabilizing transformations? Give some examples with uses.
ii) Find the test statistic for testing the equality of variances of two normal population by using logarithmic transformation.
iii) Derive the Pearsonian Chi-square test statistics for homogeneity and independence of attributes, clearly mentioning the null and the alternative hypotheses. [3+4+3]

d) i) Define Likelihood Ratio (LR) test.
ii) Find the LR size-α test statistic for testing the equality of means of two normal population when their variances are unknown.
iii) A store manager wants to see whether there is a relationship between the age of the employee and the number of sick leave they take each year. The sample data are given below. Write the test statistic and test at 5% level of significance if there is any relation between the age and number of sick leave taken.

Age (Years)	18	26	39	48	53	58
Sick Leave (Days)	16	12	9	5	6	2

(Given $t_{0\cdot025} = 2\cdot776$ for d.f 4) [2+4+4]

e) i) Consider a random sample of size n from the exponential distribution with p.d.f. :

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, x > 0 \\ 0 \quad , x < 0 \end{cases}$$

Find the maximum likelihood estimator for λ.

ii) State Cramar - Rao inequality.

iii) Obtain the minimum-variance-bound estimator for μ on the basis of a random sample of size n from $N(\mu, \sigma^2)$ population, where σ^2 is known. [5+2+3]

f) Let $(x_1, x_2, ..., x_{n_1})$ and $(y_1, y_2, ..., y_{n_2})$ be two independent random samples. Describe, in brief, the method adopted in Wald-Wolfowitz run test for testing the hypothesis the two samples come from the same distribution.

Show how the theory of runs may be used to test for the randomness of a sample. [6+4]